Stable Electromagnetic Levitation without Major Control Efforts

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Table of Contents

Table of Figures

Appendix

Context

The melting temperature of any material is effectively constant for macroscopic particles. As the size of particles is reduced to the order of a nanometer (10^{-9} meter), however, there is a sharp decrease in the melting temperature. In the case of gold particles, while the normal melting point is 1337K, nanometer sized particles melt at temperatures as low as 600K. Nanoparticles of gold can therefore be melted together with relatively low energy input. Thanks to the excellent electrical conductivity of gold, a line of gold nanoparticles sintered together forms a conductive wire if the deposition of these lines is precisely controlled. This strategy provides a new method of producing circuitry.

Current efforts at the Laboratory for Thermodynamics in Emerging Technologies (LTNT) at the Federal Institute of Technology Zurich, Switzerland (ETHZ) use a deposition system much like an ink jet printer. The “ink” is a solution of gold nanoparticles suspended in a carrier fluid, while the “paper” can be any number of materials thanks to the low melting temperatures. For precisely controlled operation of such a device, however, the hydrodynamic properties of the solution have to be known.

The solution behaves as a non-Newtonian fluid, with a decreasing viscosity as shear rate is increased. There exist devices to measure non-Newtonian viscosities, but they require a relatively large sample volume. Given that the solution is expensive, it is advantageous to waste as little of it as possible. Therefore a rheometer that measures the viscosity of a sample volume in the microliter scale is being developed. The basic principle of the rheometer follows the example of one produced by Tran-Son-Tay et al[1] in 1984. In Tran-Son-Tay’s device, a small sphere is suspended in a cylinder filled with the fluid by an electromagnetic solenoid with a direct current input. Meanwhile a second solenoid is given a sinusoidal alternating current input, which causes the sphere to follow a sinusoidal path of the same frequency but with a phase shift. This phase shift can be used to calculate the viscosity.

A problem with this design is that the levitating solenoid only produces a force exactly equal to the weight of the sphere at the equilibrium point. Any deviation from this point causes the force to change. Additionally, the system is unstable: moving the sphere towards the solenoid causes it to be pulled more strongly, while moving away from the solenoid weakens the force. The goal of this project, then, is to devise a solenoid design that provides a constant compensation for gravity over a finite range, not just at a point. Such a solenoid would have further applications than just the project at hand, such as zero gravity experiments or low friction electromagnetic bearings.

Electromagnetic Theory

Maxwell's four equations of electromagnetism are the foundations of electromagnetic theory. Two are relevant to the situation at hand.

$$\nabla \bullet B = 0 \qquad\qquad \nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

B = magnetic field (teslas)

E = electric field (volts/meter)

J = electric current density (amperes/meter^2)

μ_0 = 4πe-7 (newtons/ampere^2), magnetic permeability of free space

ε_0 = 8.854e-12 (coulomb^2/newton-meters), electric permittivity of free space

The first equation says that the total flux of magnetic field lines must always be zero. In other words, there are no point sources of magnetic field or magnetic monopoles. Any magnetic field line must therefore be a closed loop.

For the case of no electric field and a constant current density J, the second equation above gives a magnetic field with a constant curl about the wire:

$$\nabla \times B = \mu_0 J \quad \text{or} \quad B = \frac{\mu_0}{2\pi r} I \times \hat{r}$$

With r defined as the radial distance from the wire. The magnetic field lines from such a field run perpendicular to the current as follows:

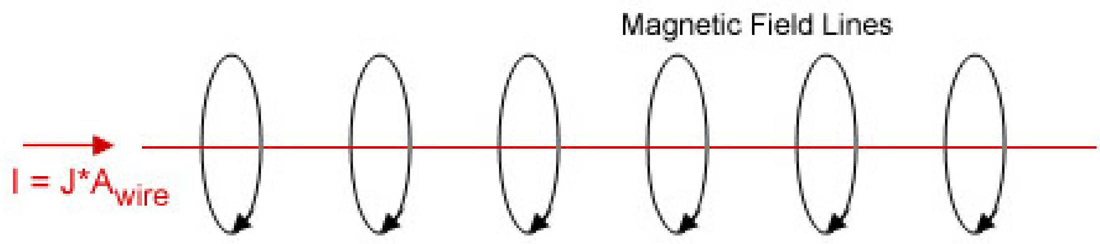


Fig. 1: Field around a straight current carrying wire

Similarly, a circular loop produces field lines which travel through the loop. Inside of an infinitely large stack of such loops – an infinite solenoid – the field lines would be aligned along the axis of the solenoid. In reality, of course, the solenoid cannot be infinitely long, and according to Maxwell's equation, the field lines must be closed loops, so there is increasing curvature of the field lines towards the ends of the solenoid as they begin to wrap back around upon themselves. This curvature will be important later, as the force exerted upon a body in a magnetic field depends on both the magnitude and gradient of the magnetic field.

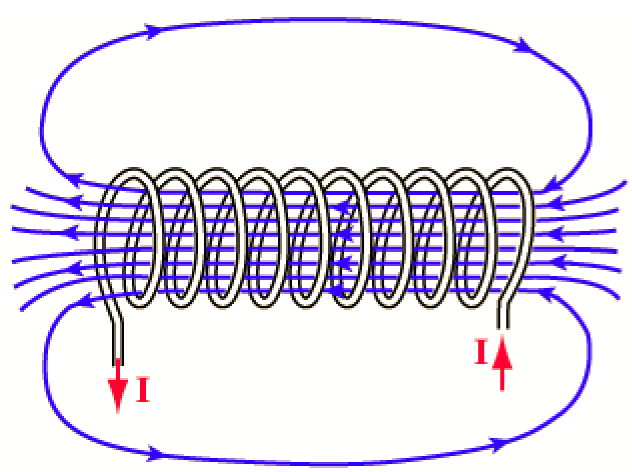


Fig. 2: Finite Solenoid

Image Credit: Hyperphysics, Georgia State University

Note that in figure 2 the middle field lines would also loop back around to form closed loops, but this has been excluded for simplicity.

Magnetisation

When a body is placed in an external field **B**, it develops a magnetisation **M** and induced magnetic field **H**, such that

$$B = \mu_0 \mu_r H \qquad M = \chi_m H \qquad \mu_r = 1 + \chi_m$$

x_m = magnetic susceptibility (zero in vacuum)

μ_r = relative magnetic permeability (one in vacuum)

All materials have a non zero value of x_m. Materials commonly thought of as non-magnetic simply have a very small value (which is usually negative), rendering the induced magnetic field **H** and any magnetic effects generally unnoticeable. So called diamagnetic materials have $x_m < 0$, while paramagnetic have $x_m > 0$. In general, paramagnetic effects are much stronger than diamagnetic. An applied magnetic field will pass through a vacuum undisturbed, with the same strength as it previously had. If a diamagnetic material is placed in this field, it will be magnetized in the opposite orientation as the applied field, creating an opposing field that essentially weakens the applied field. A paramagnetic material develops a field aligned with the applied field and enhances its strength. Indeed, paramagnetic materials and especially ferromagnetic materials – a special class of strongly paramagnetic metals which retain magnetisation after the applied field is removed – are used to enhance field strength in engineering applications.

Magnetic Force

The magnetic potential energy of a magnetic moment **M** in a field **B** is:

$$U = -\frac{1}{2} M \bullet B$$

Expressing **M** in terms of **B**, the above equation becomes:

$$U = -\frac{\chi_m}{2\mu_0\mu_r}B^2$$

The force exerted by the magnetic field is equal to the opposite of the gradient of potential **U**.

$$F_B = -\nabla U = \frac{\chi_m}{2\mu_0\mu_r}\nabla B^2$$

Assuming that the test sphere is placed along the axis of symmetry of the solenoid, and that the inner radius of the solenoid is much larger than the diameter of the sphere, the gradient can be approximated by the substantial derivative in the z direction.

$$F_B = \frac{\chi_m}{2\mu_0\mu_r}\nabla B^2 \approx \frac{\chi_m}{2\mu_0\mu_r}\frac{\partial}{\partial z}B^2 \approx \frac{\chi_m}{2\mu_0\mu_r}\frac{d}{dz}B^2$$

Since we are seeking a constant force F_B, we can rearrange the above equation and integrate over z.

$$B(z) = \left[\frac{2\mu_0\mu_r F_B}{\chi_m} z\right]^{1/2}$$

We now have an expression for the magnetic field **B** that will exert a constant force F_B in the z direction on a material with properties μ_r and x_m. It is important to remember, however, that this expression is only valid for consideration of forces along the axis of symmetry.

Solenoid Design for Constant Force Response

Given a solenoid with an axis of symmetry z, is it possible to adjust the field produced by the solenoid by varying radius and current to produce a constant force response over some range of z? Since the effective current can be varied by varying the number of coils, we define inner and outer radius functions $r_i(z)$ and $r_o(z)$, where the number of coils at a given z is equal to integer(($r_o(z)$ – $r_i(z)$) /

t), t being the thickness of wire used. The magnetic field along the axis of symmetry from one circular coil of radius r with current I is given by:

$$B_{coil}(z - z_i) = \frac{\mu_0 I r^2}{2(r^2 + (z - z_i)^2)^{3/2}}$$

where z is the distance from the plane of the coil, situated at z_i. Modeling the solenoid as a stack of such current loops results in:

$$B_{solenoid}(z) = \sum_i \sum_j B_{coil,ij}(z - z_i) = \sum_i \sum_j \frac{\mu_0 I {r_j}^2}{2({r_j}^2 + (z - z_i)^2)^{3/2}}$$

The first sum calculates the contributions from each loop that exists at a given height z_i. The second sum adds the contributions of each set of coils over all positions z_i to give the total magnetic field. Now, to convert to integral form, make the following transformations:

$$\begin{aligned} z &= tn_v & \qquad r &= tn_r \\ dz &= tdn_v & \qquad dr &= tdn_r \\ dn_v &= dz/t & \qquad dn_r &= dr/t \end{aligned}$$

where n_v is the number of coils in vertical direction z, and n_r is the number of coils in the radial direction r. Finally,

$$B(z) = \int_0^L \int_{r_i(z_0)}^{r_o(z_0)} \frac{\mu_0 \cdot I \cdot r^2}{2\left[r^2 + (z - z_0)^2\right]^{3/2}} dn_r dn_{z_0}$$

$$= \frac{\mu_0 \cdot I}{2t^2} \int_0^L \int_{r_i(z_0)}^{r_o(z_0)} \frac{r^2}{\left[r^2 + (z - z_0)^2\right]^{3/2}} dr dz_0$$

Setting this equal to the expression derived earlier for B(z) will theoretically yield the solution. However, the above expression has two unknown functions, $r_i(z_o)$ and $r_o(z_o)$. Unless there is another way to constrain the geometry of the system, the only way to solve this equation is to dictate one of the radius functions and solve for the other.

$$\frac{\mu_0 I}{2t^2}\int_0^L\int_{r_i(z)}^{r_o(z)}\frac{r^2}{(r^2+z^2)^{3/2}}drdz=\left[\frac{2\mu_0\mu_r F_B}{\chi_m}z\right]^{1/2}$$

Approximate Solution

Previously, an expression for the magnetic force was derived, under the assumption that only axial forces are considered, in the following form:

$$F_B=\frac{\chi_m}{2\mu_0\mu_r}\frac{d}{dz}B^2 \quad \text{or} \quad F_B = C_1\frac{d}{dz}B^2$$

where C_1 is a constant. Now, given that we are seeking a constant force F_B,

$$\frac{F_B}{C_1}=\frac{d}{dz}B^2 \quad \text{integrating and rearranging,} \quad \left[\frac{F_B}{C_1}z\right]^{1/2}=B$$

Thus, B must be proportional to the square root of z. Since B is also directly proportional to I and I is proportional to n_r, n_r should be proportional to the square root of z as well. The argument goes like this:

$$n_r \propto I \propto B \propto z^{1/2} \quad \text{thus} \quad n_r \propto z^{1/2}$$

Therefore, holding r_i constant and making r_o a function proportional to $z^{1/2}$, it is theoretically feasible to develop a constant electromagnetic force over a finite distance. It is important to keep in mind that this distance would have to be contained within the bounds of the solenoid.

Earnshaw's Theorem

The above analysis yields a result for a constant force in the axial direction. For the levitation to be stable, however, there also needs to exist a stabilising force in the radial direction. The feasibility of this has already been analyzed by

Earnshaw in 1842, before the discovery of diamagnetism. Earnshaw was investigating stability of 1/r potentials in general, which include gravitational, electrical, and magnetic forces. For an object to be in mechanical equilibrium, the net force acting on it must be zero. For this equilibrium to be stable, the gradient of this field **F** must be negative. Therefore the Laplacian of the corresponding potential field **U** must be positive. Mathematically,

$$F = -\nabla U = 0 \quad \text{and} \quad \nabla F = -\nabla^2 U < 0$$

The Laplacian of the sum of any set of 1/r potentials is, however, equal to zero:

$$\nabla^2 \left(\sum \frac{k}{r} \right) = \nabla^2 U = 0$$

Therefore no arrangement of 1/r potentials can result in a stable field in all directions. If the field is stable in one direction (positive Laplacian) it will be unstable in another direction (negative Laplacian) so that the net Laplacian is equal to zero. The case of the trivial solution of all second derivatives of **U** being equal to zero is metastable, but impossible in reality as the first derivatives of **U** would have to be constant. Since the form of the potential is 1/r, the first derivative of the potential cannot be a constant.

Earnshaw was concerned with fields where forces are always directed towards sources of field – gravitational, electrical, and paramagnetic fields – but he wrote his theorem before the discovery of diamagnetic materials, which experience forces directed away from magnetic field maxima. Later work by Kelvin proved that diamagnetic materials could be stably levitated[3].

In the case of the field inside of a solenoid it is fairly apparent that only a diamagnetic sphere could experience stable levitation. The magnetic potential of a sphere in a magnetic field **B**, from earlier derivation, is as follows:

$$U = -\frac{\chi_m}{2\mu_0\mu_r} B^2$$

For a diamagnetic material, x_m is negative, while for a paramagnetic material, x_m is positive. For a solenoid with windings at a radius of .4cm, the qualitative field strength and potential in the radial direction felt by a diamagnetic and paramagnetic sphere on the axis of symmetry are shown in figure 3 below. Note that the peaks at -0.4 and 0.4cm would asymptotically approach infinity as distance from the wire approaches zero. Only in the case of the diamagnetic sphere is the concavity (Laplacian in three dimensions) positive. As an analogy to gravitational potential, one can imagine the sphere "sitting" on the potential: in the diamagnetic case the sphere always rolls back to the center, while in the paramagnetic case the sphere will continue to roll farther away.

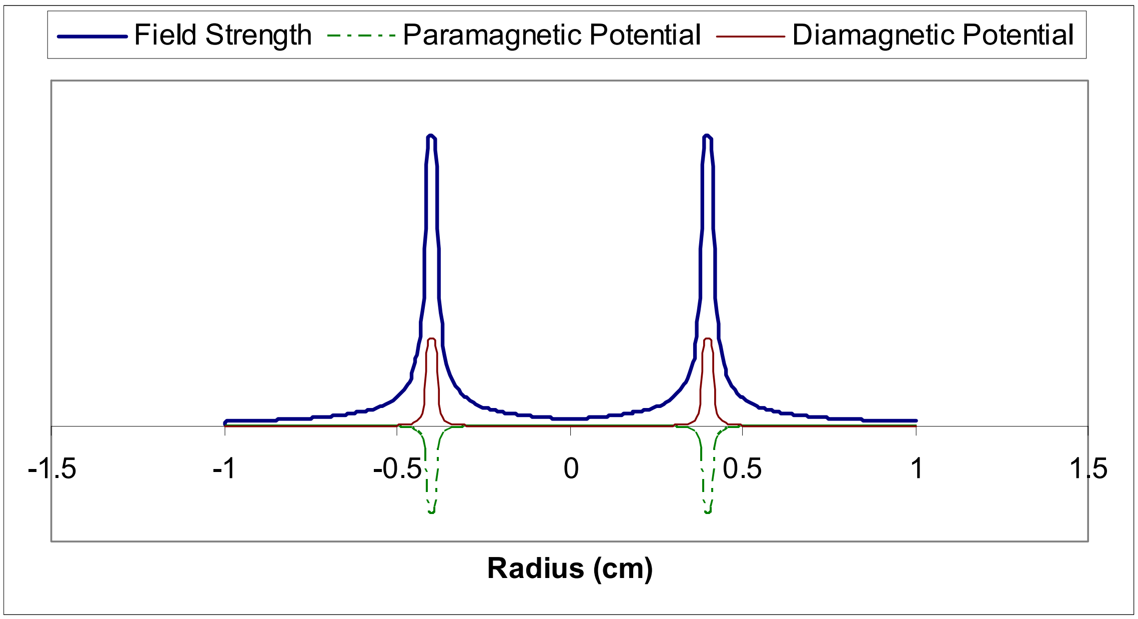


Fig. 3: Diamagnetic vs Paramagnetic Potential

Diamagnetic Stabilisation

Unconditional stable magnetic levitation, then, is possible only with a diamagnetic levitating body. In fact, this phenomenon has already been studied at length by researchers at the University of Nijmegen, who have successfully levitated graphite, water droplets, and even a living frog.[2] While graphite and water are paramagnetic, it was the water content in the frog that allowed it to be levitated. Apparently the frog was not harmed in any way by the levitation.

The strongest ordinary diamagnetic materials are bismuth and graphite, which have a value of magnetic susceptibility x_m of approximately -170e-6. Since this is not exceptionally strong, stable levitation of even a very small sphere (the rheometer requires a sphere of diameter on the order of 1mm) would require a very large solenoid. Indeed, the solenoid used at the University of Nijmegen was capable of magnetic fields of more than 10 Tesla. In our case, such a solenoid would be prohibitively expensive and could disrupt the other instrumentation, some of which would have to be placed inside of the throat of the solenoid.

In fact, based on the results of a brief program that models the solenoid as a stack of current loops, the dimensions of the solenoid can be estimated after fixing some variables. With a wire thickness of 1mm, an overall solenoid length of 2m (necessary to eliminate entry effects that curve field lines at the ends), an inner radius of 10mm, and a maximum outer radius of 100mm, the program estimates a solenoid of more than 113,000 windings. Even so, it would need 10kA of current to produce enough force to levitate a graphite sphere.

At the wider base of the solenoid the field strength would exceed 1000 Teslas. The reason our solenoid would have to be so much more powerful than the one used at the University of Nijmegen is that ours intends to support the weight over the entire length of the solenoid. For stable levitation at one specific point, the ideal placement of this point is just outside one end of the solenoid, where the rapid change in field strength produces a large gradient and a correspondingly large force. Within the solenoid the attainable gradients are much smaller.

A copy of the program and its output plots are provided in the appendix.

Controlled Modulation of Sphere

The proposed design uses a control system and one solenoid to modulate a ferromagnetic ($x_m > 0$) sphere. In principle, a sinusoidal current is fed to the solenoid, which – assuming a linear system – results in a sinusoidal output position of the sphere following a sinusoidal path of the same frequency, with an adjusted amplitude and a phase shift. In our case, there will be a second phase shift that comes from the hydrodynamic drag force exerted upon the sphere. Measuring the total phase shift between the expected position and the actual position, and subtracting the known phase shift from the control system, the viscous phase shift can be calculated. This phase shift yields the viscosity.

The assumption of a linear system is a difficult one. In reality the force is proportional to the square of the solenoid current, while it is proportional to the inverse square of the distance. Since the movement away from the equilibrium position will be fairly small, we will approximate both dependencies as linear as follows:

$$F_B = F_{equil} + K_z \delta_z + K_I \delta_I$$

where constants K_z and K_I are defined in the following manner:

$$K_z = \frac{\Delta F_B}{\Delta z}\Big|_{I=const} \quad \text{and} \quad K_I = \frac{\Delta F_B}{\Delta I}\Big|_{z=const}$$

Thus the force is assumed to be the sum of the force at the equilibrium position and linearized contributions by variations in position and current. Since we can only input a sinusoidal current with a conventional power supply, the control system will use the linearized constants as the basis for converting a sinusoidal input current into a sinusoidal output force. A direct sinusoidal input current to the solenoid would not provide a sinusoidal output force, but would exert excess force when the ball is closer and insufficient force when the ball is farther.

Mathematically, if we assume a sinusoidal current and position as follows

$$I = I_0 \sin(\omega t) \quad \text{and} \quad x = x_0 - \hat{x}\sin(\omega t - \phi)$$

and write Newton’s Second Law for the forces acting on the sphere we get:

$$F = m\ddot{x} = mg - K_S\left(\frac{I^2}{x^2}\right) - K_\eta \eta \ \dot{x}$$

where η is the viscosity. Substituting in the functions I and x yields:

$$m\ddot{x} = mg - K_S\left(\frac{I_0}{x_0}\right)^2\left[\frac{\sin(\omega t)}{1-\frac{\hat{x}}{x_0}\sin(\omega t-\phi)}\right]^2 - K_\eta \eta \ \dot{x}$$

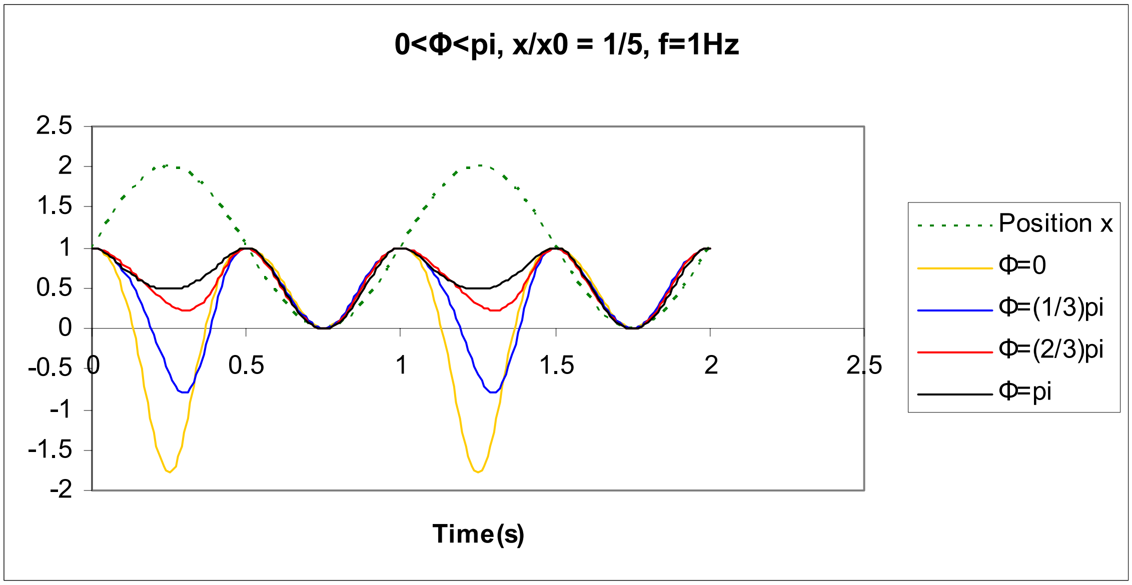
The equilibrium force should cancel gravity, so we choose I_0 and x_0 such that $Ks(I_0/x_0)^2 = mg$ and it follows that

$$m\ddot{x} = mg\left(1-\left[\frac{\sin(\omega t)}{1-\frac{\hat{x}}{x_0}\sin(\omega t-\phi)}\right]^2\right) - K_\eta \eta \ \dot{x}$$

For a sinusoidal input current to correctly account for gravity, the function in the parentheses (hereafter referred to as f*) would have to be symmetrical about the equilibrium position x_0.

$$1-\left[\frac{\sin(\omega t)}{1-\frac{\hat{x}}{x_0}\sin(\omega t-\phi)}\right]^2 = f\left(\omega t, \frac{\hat{x}}{x_0}, \phi\right) = f^*$$

Unfortunately it is not. Figure 4 below shows plots of f* with varying values of x/x_0 and Φ. Frequency and therefore angular frequency does not affect the shape of the curve and so has been held constant at a value of 1Hz. Note that while a larger phase shift and the condition $x \gg x_0$ – both of which will occur in the rheometer – help to correct asymmetry in f*, it is nevertheless symmetrical about a different average value than intended and displays a "pseudo-frequency" twice that of the position function.



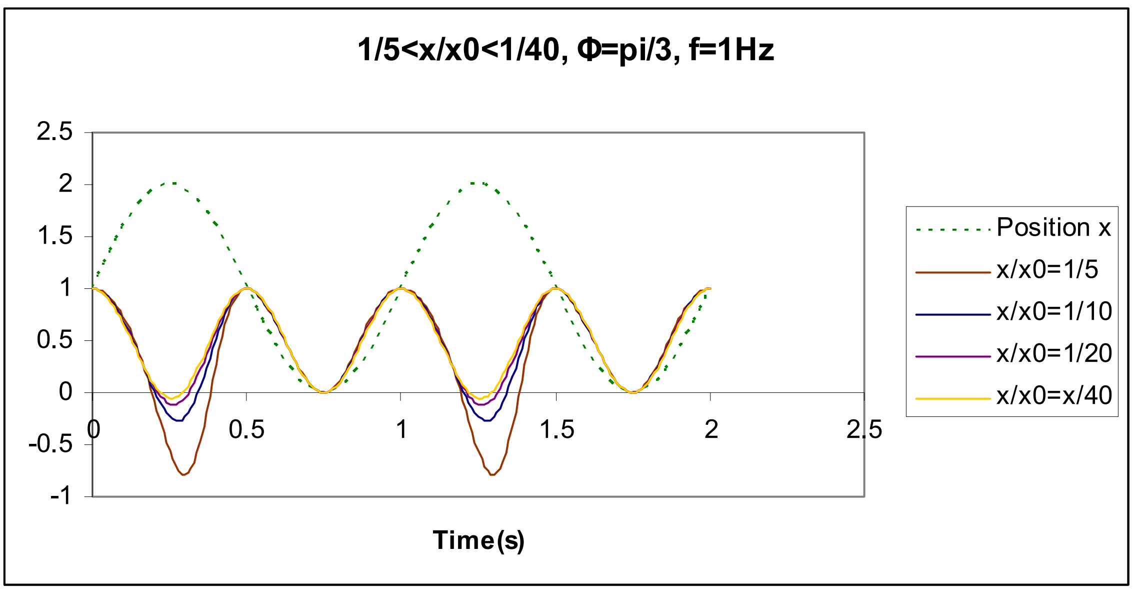


Fig. 4: Function f with Sinusoidal Current and forced Sinusoidal Position*

Therefore a control system will have to be employed to transform the sinusoidal current input. Such a complex transformation can only be satisfactorily performed with an analog control circuit. Therefore one will have to be designed and built for this application.

Note also that the solenoid can only exert an upwards directed force on the ferromagnetic sphere. At this point it is worthwhile to investigate whether a second solenoid positioned below the sphere will be necessary. The position function of the sphere will be of the form

$$z(t) = A\sin(\omega t)$$

where A is the amplitude and ω is the angular frequency. Therefore,

$$\frac{dz}{dt} = Aw\cos(\omega t) \quad \text{and} \quad \frac{d^2z}{dt^2} = -A\omega^2\sin(\omega t)$$

The maximum acceleration felt by the ball is therefore simply $A\omega^2$. In our case, A will be smaller than 1mm. Substituting 1mm for A and setting the maximum acceleration equal to the acceleration of gravity then results in a maximum possible angular frequency of 99rad/s without a second solenoid. Since the rheometer will operate in frequencies between π and 4π rad/s – well below this maximum value – only the top solenoid is necessary.

Therefore the downward force can be provided by gravity. As long as the average force on the sphere is equal to the opposite of the gravitational force, however, the net effect of gravity will be compensated for as intended. Figure 5 shows a summary of the forces acting on the sphere.

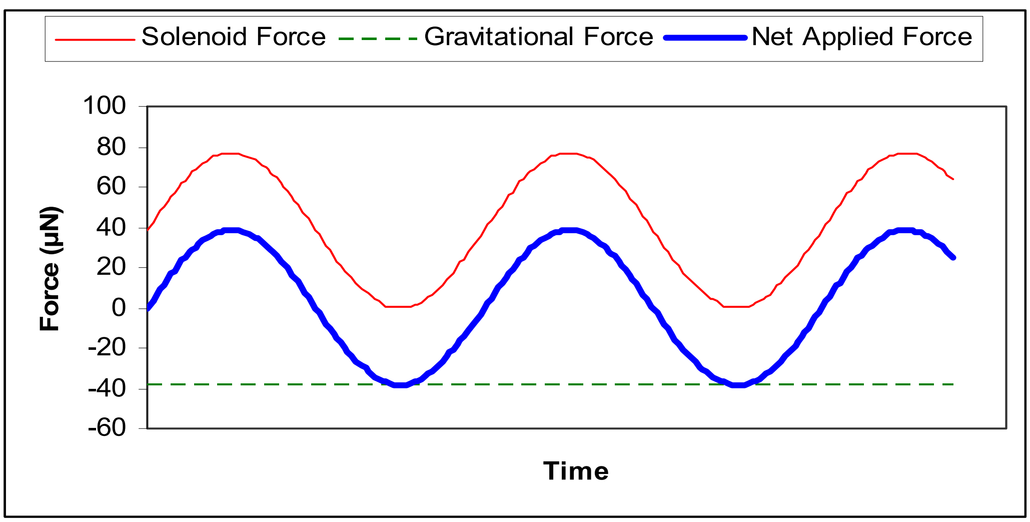


Fig. 5: Forces acting on Sphere

Since the intent is to apply a sinusoidal force to the sphere with a mean value of zero, but gravity is also acting on the sphere, the solenoid applies a two component force:

$$F_B = mg + mg\sin(\omega t) = mg\left(1 + \sin(\omega t)\right)$$

The maximum possible downward force is simply -mg, therefore the upper limit of force applied by the solenoid is set equal to 2mg for symmetry. The real device will not operate with accelerations as high as that of gravity, but as the maximum achievable acceleration it provides a convenient reference value. In order to estimate the magnitude of the forces we are dealing with, we assume a perfect sphere of 1mm diameter and a density of 7500kg/m^3 (typical of steel), which gives a sphere weight of 38.48μN (10^{-6} N).

Laboratory Exercise: Magnet Levitation

In order to gain some insight into the nature of the control problem at hand, an example circuit was built under the guidance of Simon Huwyler of the EEK (Professur für Elektrotechnische Entwicklung und Konstruktionen) group at ETH. The intent of the circuit was to use an analog proportional-differential (PD) controller to stabilize the suspension of a small permanent magnet in air. A copy of the circuit diagram is provided in the appendix. Much was learned from the construction of this test circuit. First of all, that a simple PD controller will likely not be sufficient in the rheometer circuit. It is advisable in this case to use a proportional-integral-derivative or PID controller. Also, that the cooling needs of the solenoid and circuit will have to be considered in the rheometer as significant heat was produced by both.

Determination of Linearized Parameters

An analysis of two different solenoids was carried out to determine the constants K_z and K_I as defined above. Since the forces being measured are extremely small, a counterbalanced beam scale was devised as shown:

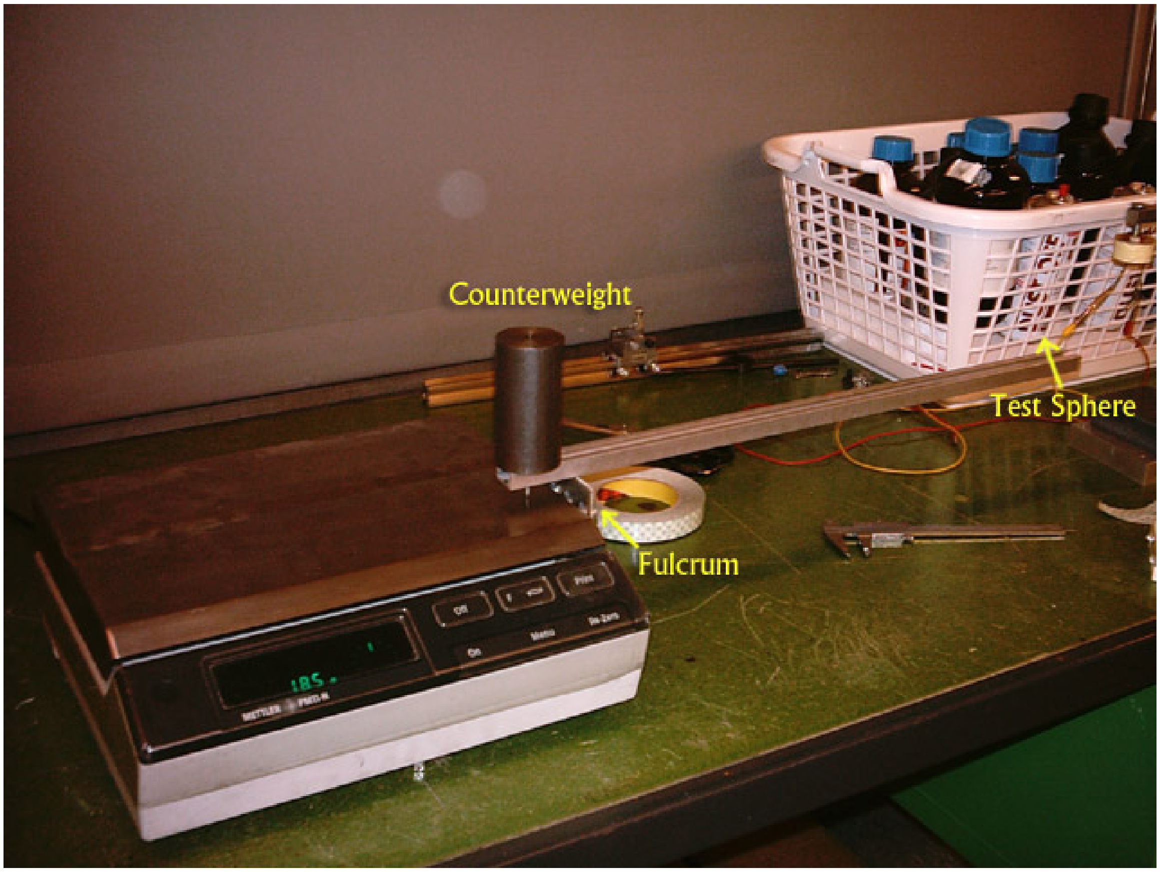
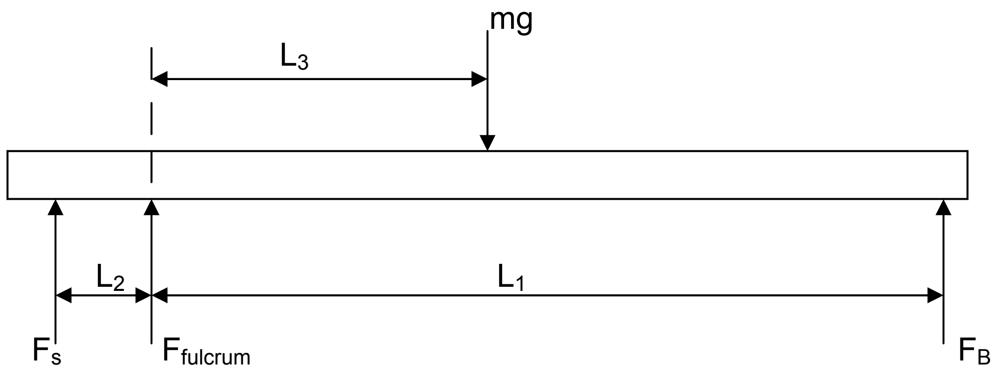


Fig. 6: Counterbalanced Beam Scale

The test sphere is glued atop a tiny glass tube. This method was used because it restrains the motion of the sphere in all directions and does not disrupt the field, because the magnetic susceptibility of glass is very small (on the order of 10^{-6}).

Because the fulcrum is placed very near to the scale, the force applied to the test sphere by the solenoid at the opposite end will be "amplified" and forces of small magnitude can be more easily measured. Another advantage of this system is that it ensures the applied field from the solenoid does not affect the scale, which employs its own electromagnetic solenoid to keep the table of the scale level. This is important for our application as it ensures that the position of the ball does not change as the force is increased, allowing us to control position and current separately. Schematically, the system looks like this:



Note that only net forces are shown. For example, at the left end the counterweight is acting downwards and the scale is applying a force upwards. Since only the changes in force will be used in the analysis we can resolve the two forces as one net force F_s. Similarly, the net force resulting from the weight of the test sphere and tube, as well as the magnetic force applied by the solenoid, are resolved into F_B. A torque balance about the fulcrum yields:

$$F_B = F_s \frac{L_2}{L_1} + mg \frac{L_3}{L_1}$$

From this equation the amplification effect is clear. The sensitivity of the scale has effectively been increased by the ratio L_1/L_2, or in the case of the experiment about 11.5 times. Since the scale reads in mass (grams) and not in force, F_s is replaced by m_s*g in the actual calculations.

In experiments of this type, there is always some uncertainty in the calculated values which results from uncertainty in the measured values. This uncertainty, called μ, can be quantified as follows:

$$\mu_{F_B} = \left[\left(\frac{\partial F_B}{\partial L_1} \mu_{L_1} \right)^2 + \left(\frac{\partial F_B}{\partial L_2} \mu_{L_2} \right)^2 + \left(\frac{\partial F_B}{\partial F_S} \mu_{F_S} \right)^2 \right]^{1/2}$$

$$= \frac{L_2}{L_1} \left[\left(\frac{F_B}{L_2} \mu_{L_1} \right)^2 + \left(\frac{F_S}{L_2} \mu_{L_2} \right)^2 + \mu_{F_S}{}^2 \right]^{1/2}$$

In this particular case, μ_{FB} = .00182N. Remembering that in the control system of the viscosity measurement device, the maximum applied force is 2mg = 76.96μN, this is an unfortunate level of uncertainty. There are various ways to potentially improve the setup. A more accurate scale would decrease the term μ_{FS} while the use of a calipers as opposed to a ruler would decrease the terms μ_{L1} and μ_{L2}. Decreasing the L_2/L_1 ratio would also increase the sensitivity of the scale to changes in applied force, so intuitively it should be made as low as possible. Mathematically, the term L_2/L_1 is a multiplier in the uncertainty term, so increasing the sensitivity of the device in this way corresponds to decreasing the uncertainty in a linear manner. However, as the beam increases in length it starts to act as a cantilever with its own weight bending it downwards at the fulcrum. With the modest L_2/L_1 ratio of 11.5 employed here, the beam was assumed to be perfectly rigid, but a higher value would mean that this effect would have to be accounted for as another source of error. Additionally, counteracting this bending would require a stronger beam material, which would necessitate a larger counterweight and a stronger fulcrum.

Experiments

In order to determine the linearized constants and also to learn something about the characteristics of electromagnetic solenoids, two solenoids were tested. Descriptions of the solenoids were as follows:

Solenoid 1		**Solenoid 2**	
Windings	500	Windings	3700
Inner Radius	5mm	Inner Radius	4mm
Outer Radius	20mm	Outer Radius	13.75mm
Height	20mm	Height	65mm

Since both of the solenoids were sealed, it was not possible to measure the thickness of the wires in each solenoid. As a visual estimation, the wire in solenoid 1 was on the order of 1mm while the wire in solenoid 2 was on the order of 0.2mm. This allowed solenoid 1 to handle more current without overheating.

A holder for the solenoid was devised from a micrometer so that the distance could be manipulated precisely. A power supply capable of delivering current at an accuracy of +/- .01 amperes was used to power the solenoid.

Experiment 1: Solenoid Force

The first experiment performed sought a characterization of the force as a function of current and distance from the solenoid. The solenoid was therefore tested throughout a range of current at various distances from the scale. Raw data is given in the appendix, but the general form of the results was that at each distance force is approximately a quadratic function of current. Mathematical fits were made so that the dependency of force on distance could also be calculated.

To calculate the constants K_I and K_z, simply choose a desired equilibrium distance and calculate the partial derivatives of F(I,z) with respect to I and z at that point. For example, with a distance of 3mm from solenoid 1, the force is approximated by the function

$$F_{3mm} = -99.78I^2 + 544.4I \quad \text{and} \quad \frac{\partial F_{3mm}}{\partial I} = -199.56I + 544.4$$

with force in terms of μN and current in terms of amperes. Setting equilibrium force equal to the weight of the sphere yields I = .0716A. Substituting this value of I into the second equation,

$$\frac{\partial F_{3mm}}{\partial I}\Big|_{I=.0716} = K_I = 530.1\frac{\mu N}{A}$$

As for the distance constant, simply substitute the equilibrium value of I into the functions for F at 2mm and 4mm:

$$F_{2mm} = -172.4I^2 + 876.9I = 61.92\mu N$$
$$F_{4mm} = 14.23I^2 + 360.7I = 25.91\mu N$$

The value of K_z is taken to be the average of the slopes between F_{3mm} and each of the other two functions:

$$\left.\frac{\Delta F}{\Delta z}\right._{avg} = \frac{1}{2}\left(\left.\frac{\Delta F}{\Delta z}\right|_{2,3} + \left.\frac{\Delta F}{\Delta z}\right|_{3,4}\right) = K_z = -18\frac{\mu N}{mm}$$

These values can then be used as a baseline for the design of a control system to modulate the sphere. Reassuringly, the results of the experiment show that the functions are only weakly quadratic and that linearizations will give a good approximation of the actual behavior of the solenoid.

Experiment 2: Force with Positive and Negative Current

In all of the experiments, the solenoid was supported with a steel bolt. Given that steel is ferromagnetic ($x_m \approx 50$) it enhances the field strength and reduces the amount of current necessary to create a given electromagnetic force. It also poses a potential problem in that ferromagnetic materials retain some degree of magnetisation after the applied magnetic field is removed, resulting in a hysteresis loop for magnetisation **M** vs applied field **B**.

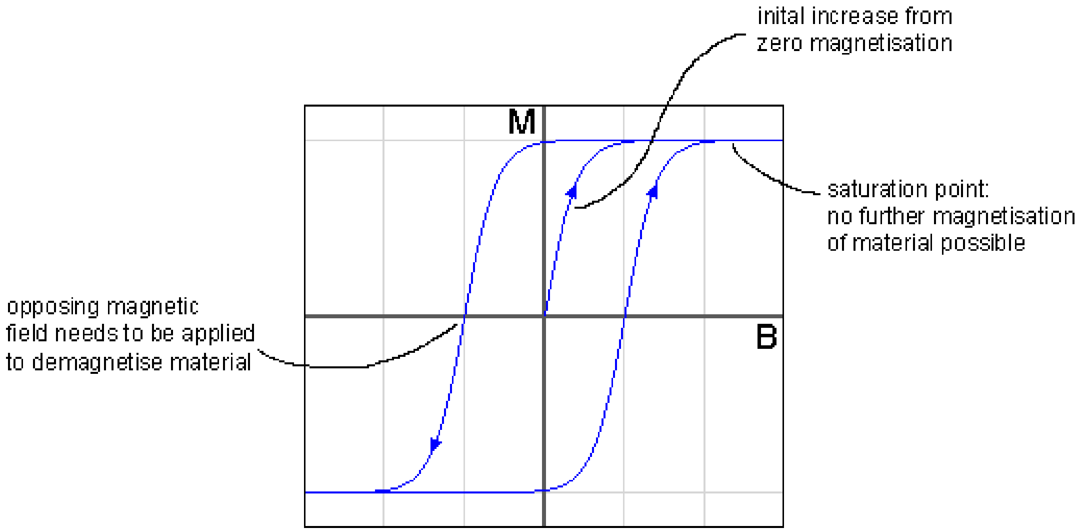


Figure 7: Hysteresis Loop

Image Credit: Wipikedia Free Online Encyclopedia

Therefore it is possible that the magnetic field acting on the ball in the previous experiment came from not only the solenoid but also the steel bolt. To investigate this effect, another experiment was performed in which the current was first increased and decreased in one voltage orientation, then the orientation switched immediately thereafter and increased and decreased again. Theoretically this procedure would encompass the entire hysteresis loop.

Note, however, that switching the direction of current flow will not change the direction of the force. Because the sphere is always magnetised in the same orientation as the net field applied to it, it will always be pulled towards field maxima. In principle, after the current changes orientation but before it is increased to the point that it demagnetises the bolt, the ball will continue to feel a force.

In the actual experiment a negative force was observed. Comprehensive raw data is again provided in the appendix. Unfortunately this probably has more to do with vibrations from adjusting the wires and power supply disrupting the balance of the beam than any physical phenomenon. It is interesting to note, however, that immediately after changing voltage orientation the current had to be increased appreciably (~1 ampere) before a change could be observed in force felt by the sphere. This is because the solenoid was applying a field opposing the field of the bolt, so that the net field felt by the sphere was relatively unaffected until the solenoid had “reset” the magnetisation of the bolt to be parallel with its own. The use of a power supply capable of switching directly from positive to negative voltage, as well as a better isolated table, could improve the accuracy of the results of this experiment.

Experiment 3: Field Strength

In addition to measurements of force, both of the solenoids were also tested for magnetic field strength. This experiment utilized a hall sensor and the same micrometer based solenoid holder. The hall sensor is a common device used to measure magnetic field strength. In principle, it simply produces a small current, which, in the presence of a magnetic field, is deflected resulting in a voltage perpendicular to the direction of current flow. This voltage is proportional to the field strength.

Modeling the solenoid as a stack of current loops, the field strength should be linearly dependent on the current. Indeed, this is the case for solenoid 1 (raw data once again provided in appendix). For solenoid 2, however, the field strength had a tendency to level off at higher current. This is probably a result of the longer steel bolt employed in supporting solenoid 2, which would initially act to enhance the field but would no longer contribute after its magnetic saturation, leaving only the solenoid which is of course comparatively weaker than the solenoid and bolt together.

In the case of solenoid 1 the bolt was shorter and therefore the ferromagnetic effects were reduced.

Conclusion

This project investigated the feasibility of an electromagnetic solenoid capable of stable levitation over a given distance without control effects. The intent was to use such a solenoid in a rheometric device for measuring dynamic viscosity. It was concluded that while such a solenoid is possible theoretically with diamagnetic levitation, it is not practical to implement. Therefore an alternate rheometer design was devised that uses a control system and paramagnetic levitation. Initial progress was made towards the completion of this control system in the form of determination of linearized constants characterizing the response of two potential solenoids. Unfortunately time restraints have prevented the rheometer from being completed at the time of this writing. It should be noted that this project was a collaborative effort and that the design of the remaining parts of the rheometer such as the distance measurement system, cooling system, and housing was done by Robert Büchel as a separate research project.

References

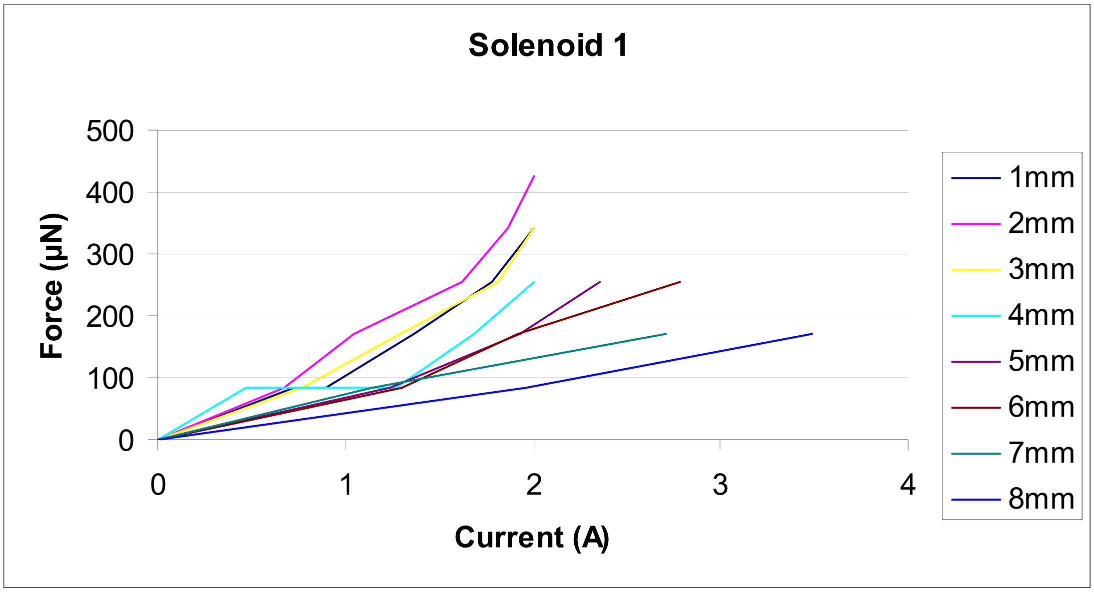
1 R. Tran-Son-Tay, B.B. Beaty, D.N. Acker, R.M. Hockmuth, Magnetically driven, acoustically tracked, translating-ball rheometer for small, opaque samples, *Rev. Sci. Instrum*, Aug. 1988, **59** (8): 1399-1404

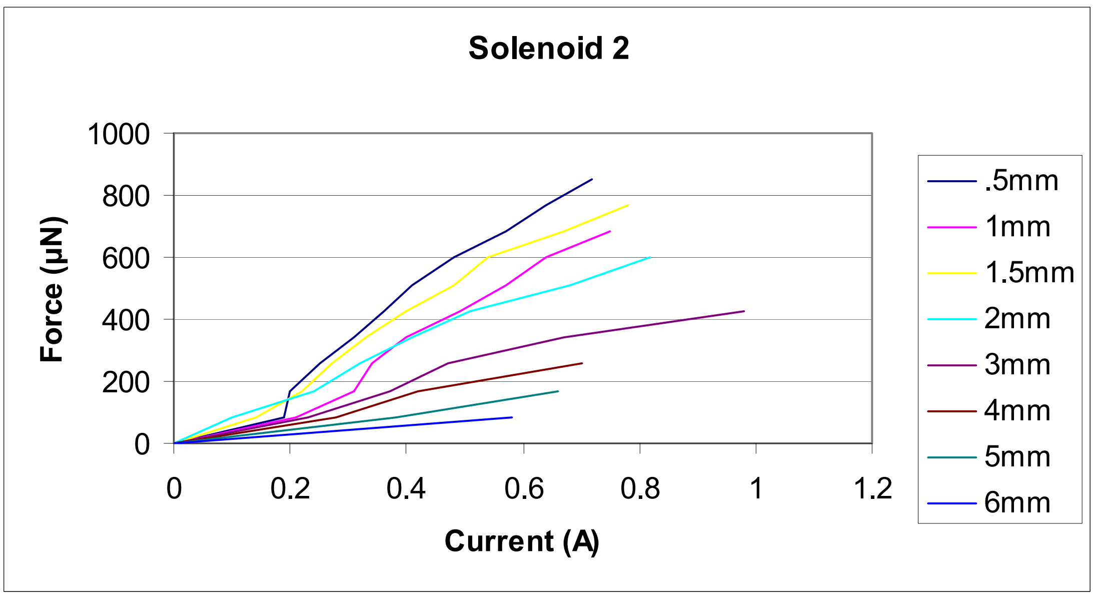
2 M.D. Simon, L.O. Heflinger, A.K. Geim, Diamagnetically Stabilized Magnet Levitation, *AM J Phys*, Jun. 2001, **69** (6): 702-713

3 W. Thompson, *Reprint of Papers on Electrostatics and Magnetism* (Macmillan, London, 1872), paper XXXIII, pp. 493-499, paper XXXIV, pp. 514-515

Appendix

Experiment 1: Solenoid Force



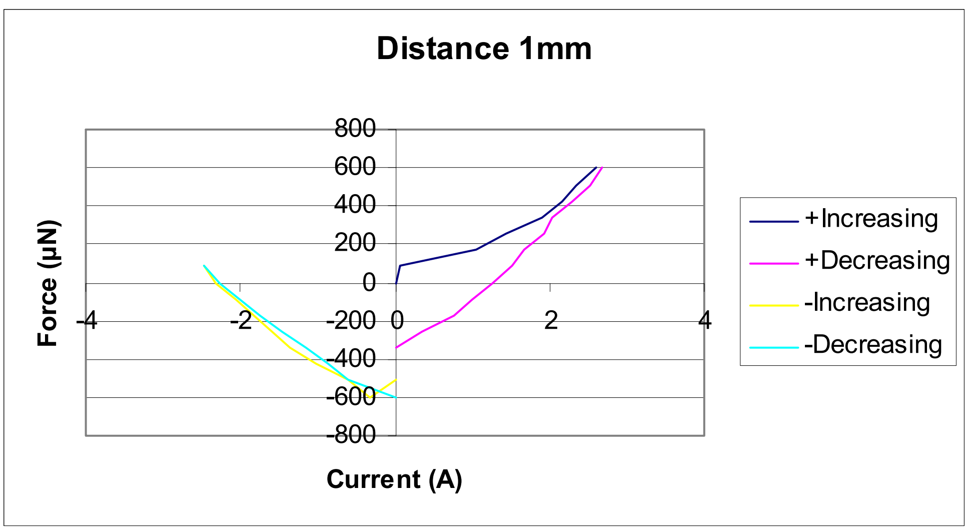


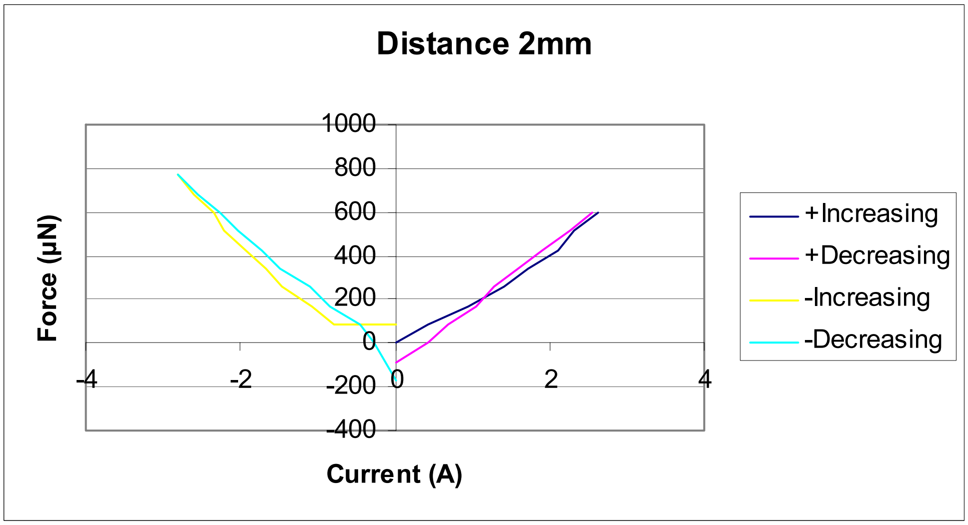
Force vs Current – Approximate Fit

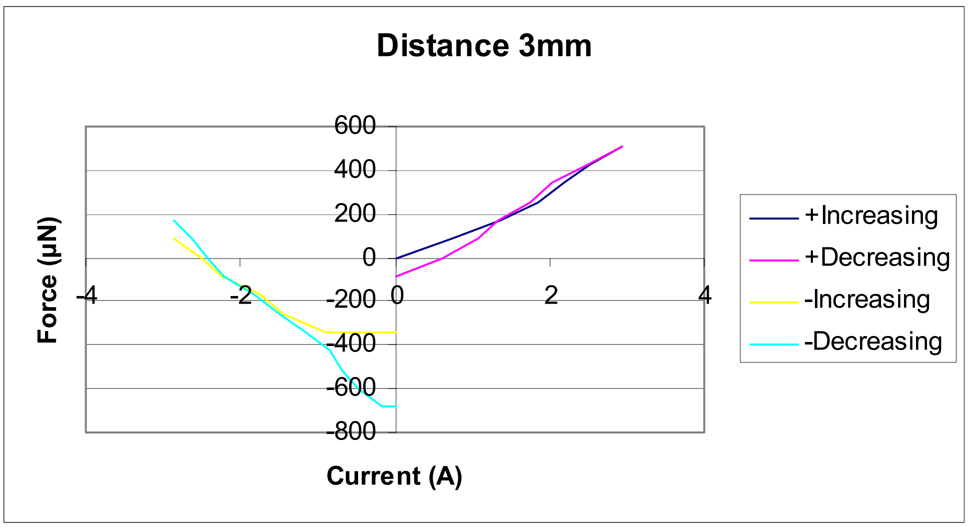
Solenoid 1 Distance (mm)	F(Ne-6) = C1*I^2 + C2*I C1	C2
1	55.74	52.04
2	56.92	85.07
3	46.87	68.91
4	32.68	52.91
5	36.81	20.42
6	13.05	57.18
7	-7.568	83.51
8	3.507	36.68

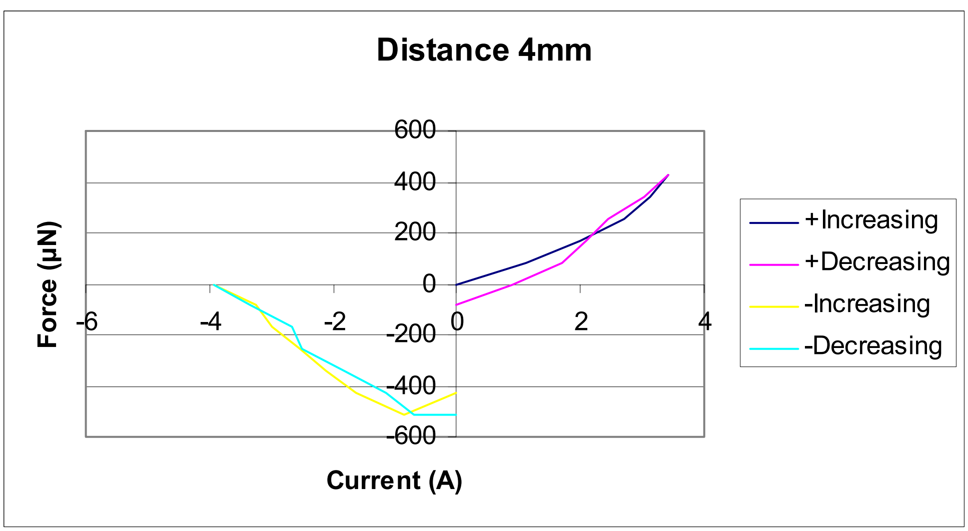
Solenoid 2 Distance (mm)	F(Ne-6) = C1*I^2 + C2*I C1	C2
0.5	359.2	976.7
1	557.7	543
1.5	21.51	1007
2	-172.4	876.9
3	-99.78	544.4
4	14.23	360.7
5	121.6	178.5
6	0	147.2

Experiment 2: Force with Positive and Negative Current



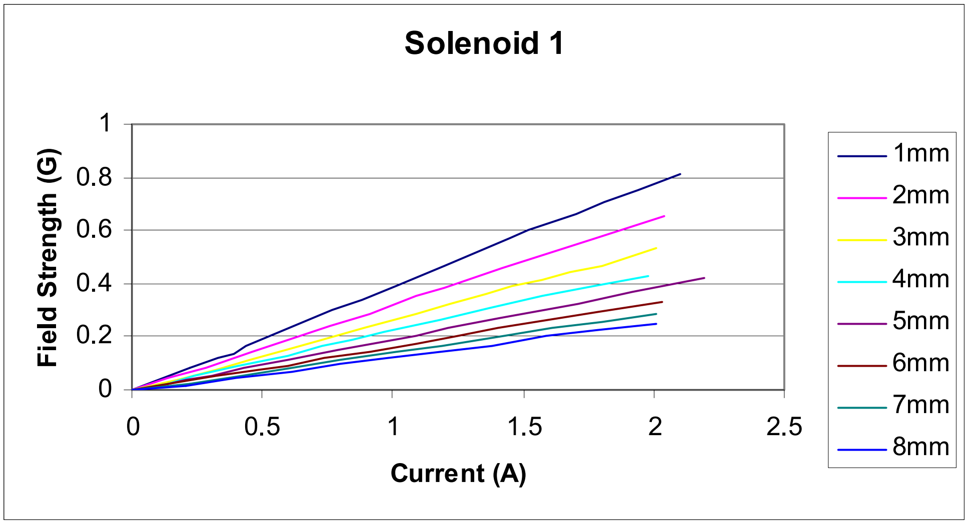

Distance 2mm
Force (µN)
Current (A)
1000
800
600
400
200
0
-200
-400
-4
-2
0
2
4
+Increasing
+Decreasing
-Increasing
-Decreasing

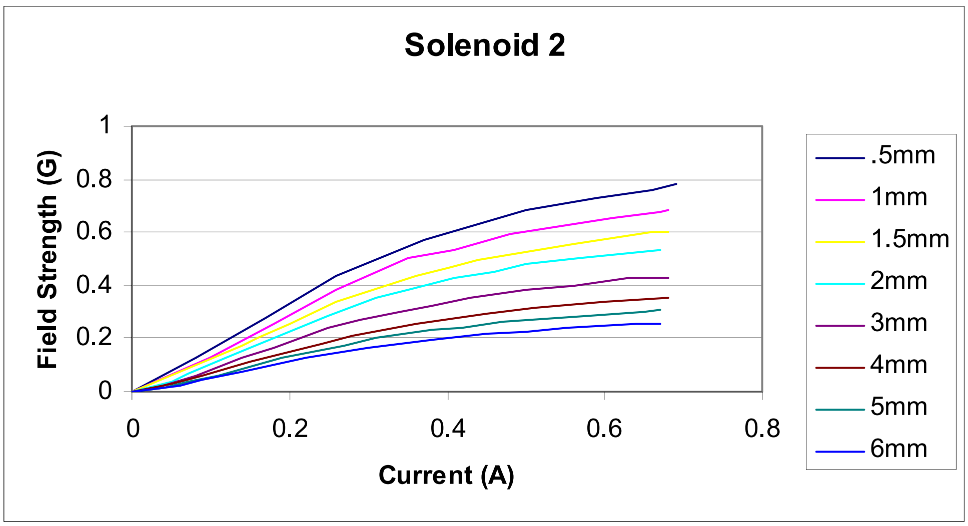

Distance 3mm
Force (µN)
Current (A)
600
400
200
0
-200
-400
-600
-800
-4
-2
0
2
4
+Increasing
+Decreasing
-Increasing
-Decreasing


Distance 4mm
Force (µN)
Current (A)
600
400
200
0
-200
-400
-600
-6
-4
-2
0
2
4
+Increasing
+Decreasing
-Increasing
-Decreasing

Peter Tennessen
LTNT, ETH Zürich

Experiment 3: Field Strength





Field Strength vs Current – Approximate Fit

Solenoid 1 Distance (mm)	B(G) = C1*I C1
1	0.3884
2	0.3198
3	0.263
4	0.2214
5	0.1907
6	0.163
7	0.1425
8	0.1237

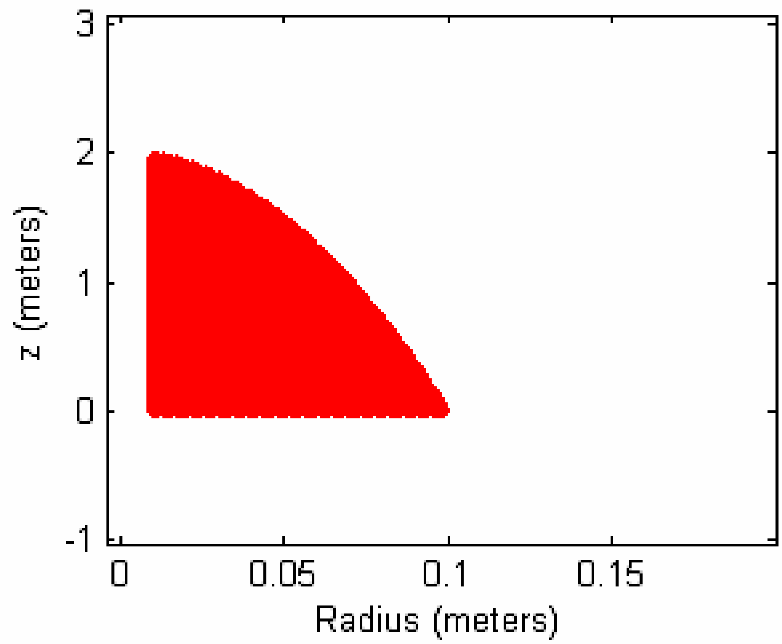
Solenoid 2 Distance (mm)	B(G) = C1*I^2 + C2*I C1	C2
0.5	-1.2061	1.9598
1	-1.0743	1.7387
1.5	-0.8973	1.5019
2	-0.7623	1.3276
3	-0.6637	1.0927
4	-0.5147	0.8762
5	-0.4584	0.7654
6	-0.3801	0.6428

MATLAB Program Solenoid.m

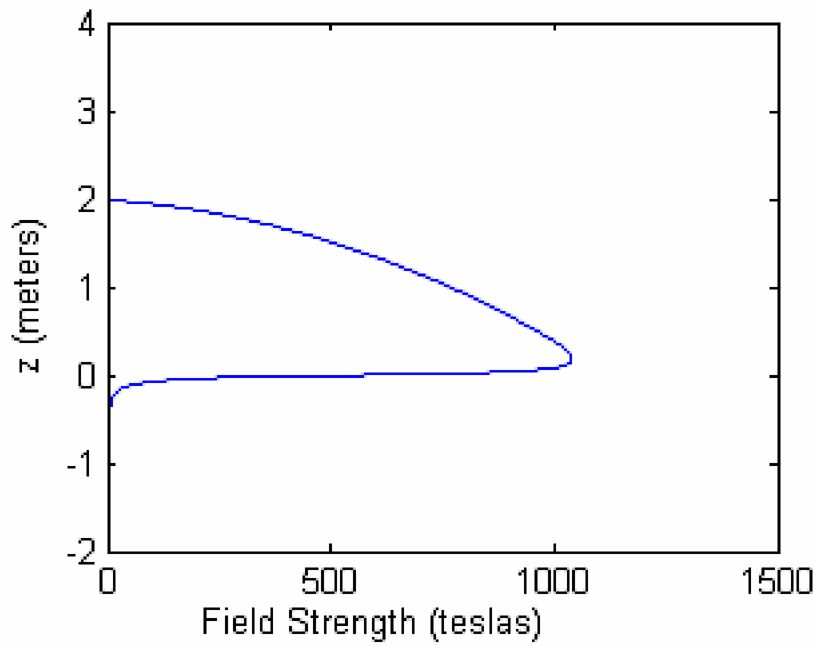
```
% this program calculates the magnetic field along the central axis of an
% axisymmetric solenoid.  user can vary the geometry (radius) and thickness
% (num of coils) at each vertical point
% program then uses field to determine force as a function of position,
% assuming force is proportional to field*grad(field) or B*dB/dz
%SI units - m, kg, N, amperes etc
%overall solenoid length L
%thickness of wire t
%cross sectional area of wire A
%length of wire LW
%basic current running through solenoid I
%solenoid coil geometry matrix S = [z position, radius]
%position matrix P from bottom to top of matrix (for plotting)
%magnetic field B = B(z)
%derivative dB/dz = gradB
%force/constant F = B*gradB
%resolution of B in z = a, normally L/1000
%shape of inner wall of solenoid r_in(z)
%shape of outer wall of solenoid r_out(z)
%axial position z array u = S(*,1)
%radius r array v = S(*,2)
L = 2.0;
t = .001;
A = (pi/4)*t^2;
LW = 0;
I = 10000;
clear S;
clear P;
clear B;
clear gradB;
clear F;
a = L/1000;
clear r_in;
clear r_out;
clear u;
clear v;
clear max;
%graphite sphere of 1mm diameter
V = 1.767e-9    %volume of test body m^3
rho = 2160      %density kg/m^3
x = -170e-6     %susceptibility, unitless
disp('press control + c to interrupt program')
%reset index i
i=1;
disp('building solenoid geometry matrix...')
disp('windings count:')
for z = L:-t:0
    %define here functions r_in and r_out for shape of solenoid
    %program then adds the number of loops n at current position z to
    %achieve desired shape, round because you can't make 1/2 a loop
    r_in = .010;
    r_out = (.10/(L^.5))*(L-z)^.5;
    %r_out = .10;
    n = round((r_out-r_in)/t);
    for j = 1:n
        S(i,1) = z;
        %radius, second term accounts for increasing radius as coils
        %overlap at same z
        r = r_in + t*(j-1);
        S(i,2) = r_in + t*(j-1);
```

```
        LW = LW + pi*2*r;
        u(i) = S(i,1);
        v(i) = S(i,2);
        i = i+1;
        if size(S,1)/1000 == round(size(S,1)/1000)
            disp(size(S,1))
        end
    end
end
%reset index i
i = 1;
disp('calculating magnetic field strength...')
for z = -L:a:2*L
    %dummy variable strength of b at particular z
    b = 0;
    for j= 1:size(S,1)
        %sum contributions to field through all coils
        b = b + ((1.256637e-6) * I * S(j,2)^2 / (2 * (S(j,2)^2 + (z-
S(j,1))^2)^1.5));
    end
    B(i) = b;
    P(i) = z;
    i = i+1;
end
%numerical differentiation of B(z), dB/dz = grad(B)
disp('calculating dB/dz...')
for i = 3:(size(B,2)-2)
    gradB(i) = (-B(i+2) + 8*B(i+1) - 8*B(i-1) + B(i-2))/(12*a);
end
%this fills in extra values of dB/dz that were out of range with 4th order
%differentiation.  simply reuses 3rd value for 1st and 2nd, 3rd last value
%for 2nd last and last.  i know it's inaccurate but at a distance L from
%solenoid the magnitude of B and slope of B are very near zero anyway
gradB(1) = gradB(3);
gradB(2) = gradB(3);
gradB(size(B,2)-1) = gradB(size(B,2)-2);
gradB(size(B,2)) = gradB(size(B,2)-2);
%multiplication of B*gradB, assumption being that constant*B*gradB = Force
disp('calculating force...')
for i = 1:size(B,2)
    F(i) = (x*V*B(i)*gradB(i))/(rho*1.256637e-6);
    max(i) = 9.8*rho*V;
end
LW
R = (1.7e-8)*LW/A
disp('power consumption')
disp((I^2)*R)
%display results in plots
subplot(2,2,1), plot(v,u,'r.')
axis([0 L -L 2*L])
xlabel('Radius (meters)')
ylabel('z (meters)')
subplot(2,2,2), plot(B,P,'b-')
xlabel('Field Strength (teslas)')
subplot(2,2,3), plot(gradB,P,'g-')
xlabel('dB/dz')
ylabel('z(meters)')
subplot(2,2,4), plot(F,P,'k-')
xlabel('Force (newtons)')
%hold on
%plot(max,P,'y-')
%hold off
```

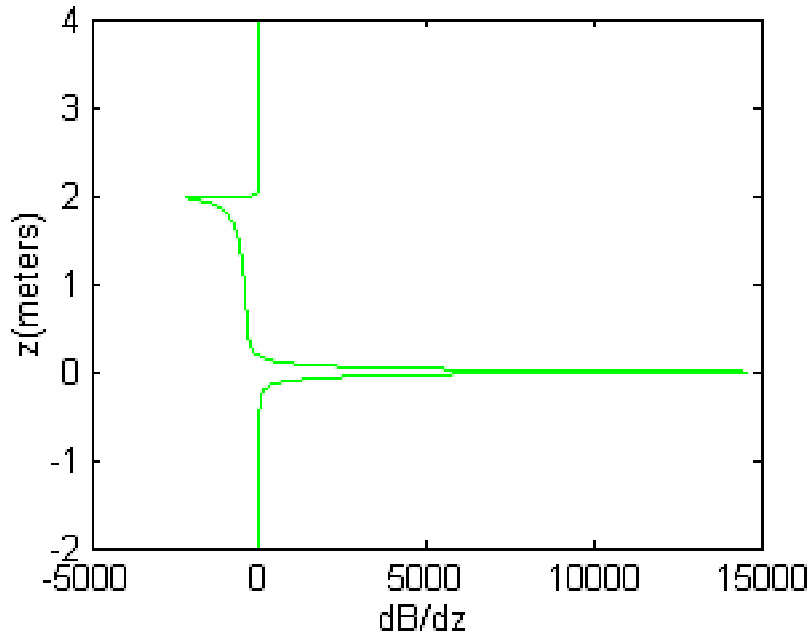
Program Output



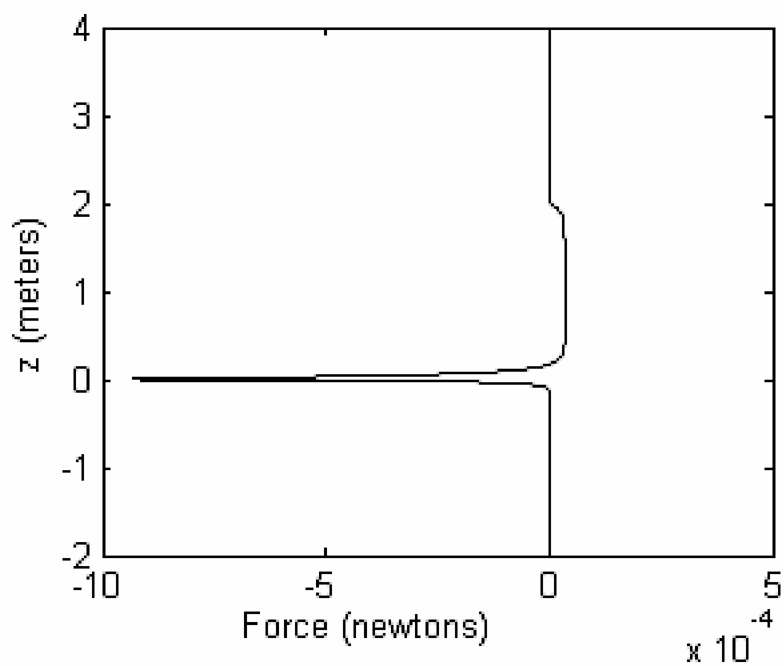
Geometry of Solenoid



Field Strength generated by solenoid



Derivative of Field Strength



Force vs Axial Position
Note that the weight of the sphere is ~37.4µN

Magnet Levitation Circuit Diagram